

ERROR PATTERNS IN ORDERING AND COMPARING FRACTIONS AMONG FOURTH-GRADE STUDENTS

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Abstract: The study aimed to identify the pattern of misconceptions and analyze their causes. In this study, a test containing fraction-based problems was administered to a sample of 60 students in grade 4. Descriptive statistical analysis showed mean, median 7 and mode at 6 making it moderate performance with a total score of 6.90. The standard deviation is 1.67, and the variance was 2.80, indicating varying students' understanding in this range of 7. Noteworthy, 52.3% of errors were reported in ordering and comparing questions signalling that the students encounter immense conceptual difficulty. Your research showed that many students struggled with conceptual understanding issues like whole number bias, incorrectly interpreting the denominator, and in using benchmarking fractions. The results demonstrate that without a strong conceptual understanding students struggle to compare fractions.

Keywords: fraction, ordering, comparing, conceptual knowledge, procedural knowledge, marginalized, fourth-grade students, whole Number Bias

Introduction

Understanding fractions serves as a fundamental building block for various concepts in mathematics that build a strong foundation for advanced mathematical skills and various sought-after careers (Tian & Siegler, 2017). Fractions play a significant role in various fields like physics, biology, social sciences, and even in several average-income professions that don't require advanced-level mathematics skills. It comprises professions like carpentry, nursing, and auto mechanics, emphasizing the practical importance of understanding fractions beyond academic settings. Fractions play a crucial role in handling real-life problems, from dividing the parts of a pizza to sharing something among fellows. Fractions help students to develop critical thinking; and problem-solving skills as well as logical reasoning (Kilpatrick et al., 2001). Here, a point needs to be clear that operational knowledge or comprehension is the symbolism that is used while performing mathematical operations. In contrast, conceptual comprehension or knowledge is an important component of operational capability in handling mathematics. (Van de Walle et.al, 2014).

One of the most widely documented challenges in fraction learning is the persistence of misconceptions among primary school students. Kukulaje et al. (2024) highlighted that primary school students demonstrated persistent misunderstandings in fraction representation and comparison tasks, specifically when the reasoning is required about the size of fractions. These misconceptions hinder students' ability to compare fractions accurately and lead to systematic errors in ordering and comparing. The general student learning Objective (SLO) of the Single Nation Curriculum (SNC) of Mathematics in Pakistan includes boosting students' abilities to understand mathematical concepts and use them in their daily lives, along with representing these concepts in different ways. However, students don't always grasp the correct concept

ts, and unfortunately, they don't emphasize or give importance to unlearning and relearning, and carry those incorrect concepts or misconceptions throughout their lives. In Mathematics, when students learn something partly or inaccurately, they face complications and these complications are reflected in their impending educational lives. These inaccurately or partly learned concepts can lead to misconceptions. Misconceptions result from incorrect application of the rule or incomplete comprehension of the concepts. The best practice to understand something that either every wrong attempt is a misconception or only a mistake is to check out the frequency and consistency of the mistake. It is understood that not having a due level of understanding of any concept and mathematical operations leads a student towards misunderstanding or misapprehension of the idea. It leads students to develop negativity towards mathematics (Resnick et al., 2016). When students begin to tackle symbolic fraction arithmetic problems, they often make errors that arise from the overgeneralization of whole number rules to fractions and the omission of crucial steps (Braithwaite & Seigler, 2020).

Whole number bias is a major contributor to these misconceptions, where students apply whole number reasoning to fractions. Many students assume that a fraction with a large denominator is greater, as they focus on the magnitude of the numbers rather than the value of the fraction as a whole. Students of primary school frequently exhibit this type of reasoning. Research studies (Caute et al., 2025; Ratnasari, 2018; Nauraini et al., 2016) that students in primary schools face difficulties, which are not merely random errors but are due to the deep-seated cognitive biases and instructional gaps. Students frequently misapply their previous knowledge and concept of whole numbers, "Whole Number Bias", counting properties to rational numbers, ignoring the relationship of the numerator and the denominators, and leading to the common "add-across" error where numerators and denominators are treated as independent integers (Powell and Nelson 2021). A critical lack of relational understanding is highlighted in research studies where students rely on rote memorization of procedures rather than acquiring the fundamental magnitude of the fraction (Hidaatullah & Zainil, 2025).

Persistence of errors in comparing fractions also points to the stability of misconceptions over time. Fikriyah et al. (2025) revealed that misconceptions in fractions are often deeply rooted and can badly affect the reasoning skills, even after instructions. This has also persisted through an approach of using errors as a way to, better identify and analyse the ways in which students think. If we talk about the error patterns in fractions, they prevent the students from understanding and working well with them. The errors arise due to weak fundamental knowledge, misconceptions and procedural learning. Literature indicates that students make mistakes in fractions because of reading errors, understanding errors, encoding errors (Schumacher & Malone, 2017), execution errors interplay with process skills and conceptual as well as transformation errors. No interest in mathematics, no perseverance in solving problems, interruption between learning approaches and knowledge not

fully acquired can produce those patterns of errors. The bad thinking processes can also make it harder for children to compare fractions in difficult cases, i.e. when the big numerator or denominator is not matching up.

In addition, the conceptual misunderstanding students encounter also makes the processes of applying suitable strategies to compare fractions more complex. In a newly-published study (Journal of Mathematical Behavior, 2022), we observed that even though students have access to several strategies—that is, find the common denominators, use the benchmarking technique, or use cross-multiplication—they are often challenged in applying these methods correctly. Most of the students just memorize the procedures with an eye on rote learning and lack understanding of the actual underlying concepts, which leads to errors. The overreliance on procedural knowledge indicates the need for instructional approaches that emphasize conceptual understanding instead of rote learning. The denominator represents the total number of parts a whole has been divided into, while the numerator represents the number of parts that have been taken or are under discussion (Mamede et al., 2005). As Lamon (1999) put it, the interpretation of the number of parts in a given unit is an interpretation that can be different from the other constructs in the sense that the equal number of parts in each unit can change according to the number of times the unit is being partitioned. The sequential portioning enables us to be precise in measuring (Clarke et al., 2008). These measurements have an opportunity to be expressed on a number line in terms of points.

Development of fraction magnitude understanding, which refers to the ability to perceive and reason about the fraction size, is another critical aspect of fraction learning. According to the research in the Journal of Mathematical Behavior (2023), understanding fraction magnitude is essential to grasp the concept of comparing and ordering correctly.

Similar to the other types of problems, comparing and ordering fractions also requires cognitive factors as there are several components that need to be integrated: the numerator, denominator, and overall magnitude. The misconceptions of the students are connected with cognitive processing and knowledge from the past (Schadl & Lindmeier, 2025). If students cannot integrate the concept well they start using oversimplified or erroneous reasoning. However, this faulty reasoning causes consistent mistakes are made, specifically in those questions with multiple fractions or complex comparisons.

Error analysis is an effective way to identify gaps in learning and to understand students' thinking. Analyses of errors can help teachers to modify or select appropriate and relevant instructional strategies to address specific errors and misconceptions. The study focuses on identifying and analysing common error patterns in ordering and comparing fractions among grade 4 students.

Research Questions

The following questions were addressed through this study

- What types of errors do students make in ordering and comparing of fractions?
- What are the most frequent error patterns observed?
- What misconceptions are reflected through these errors?

Research Design

A descriptive research design was used to analyze students' errors. A sample of 60 grade 4 students was selected. A written test consisting of ordering and comparing fractions was used to collect data.

The validity and reliability of the test were ensured before implementation. The data were analyzed using descriptive analysis.

Result Analysis and Findings

Students' responses were examined to identify and classify error patterns. The frequency and percentage of each error type were calculated to determine the common errors in fraction addition and subtraction.

Table 1

Performance of Students in Fractions

Sub-Topic	Total Responses	Correct Responses	Incorrect Responses	Error Percentage
Ordering and Comparing Fractions	900	429	471	52.3

Note: The table presents the performance of students across fraction comparison and ordering.

The analysis highlights that a total of nine hundred responses were analyzed. Of these, 471 answers were incorrect and 429 correct, yielding an overall percentage of 52.3%. Study findings showed that more than half of the responses were incorrect, this suggesting an issue with fraction operations within students.

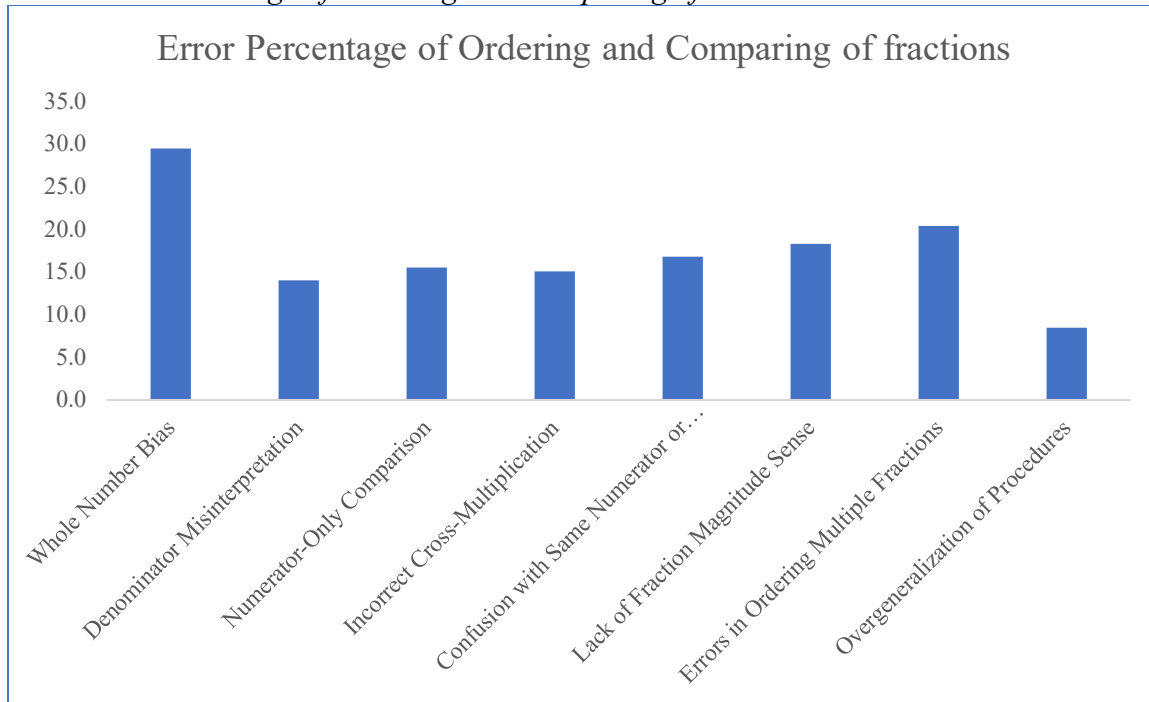
Table 2

Categorical Classification of identified Error pattern

Errors Category	Description	%
1. Whole Number Bias	Treating numerators and denominators as separate whole numbers (e.g., thinking $1/8 > 1/6$ because $8 > 6$).	29.5
2. Denominator Misinterpretation	Assuming a larger denominator means a larger fraction.	14.0
3. Numerator-Only Comparison	Focusing only on numerators while ignoring denominators.	15.5
4. Incorrect Cross-Multiplication	Errors in applying or interpreting cross-multiplication.	15.1
5. Confusion with Same Numerator or Denominator	Not recognizing patterns (e.g., same numerator $\rightarrow$ smaller denominator means larger fraction).	16.8
6. Lack of Fraction Magnitude Sense	Inability to estimate or judge size using benchmarks (like $1/2$ or 1).	18.3
7. Errors in Ordering Multiple Fractions	Difficulty arranging more than two fractions in the correct order.	20.4
8. Overgeneralization of Procedures	Applying one method (e.g., cross-multiplication) in all situations, even when it's inappropriate.	8.5

The analysis found that students often use whole-number reasoning when comparing fractions, so the most common error is: Whole-number bias (29.5%). The error pattern with the second highest identification was that on more than one fractions to order (20.4).

Figure 1: *Error Percentage of Ordering and Comparing of Fractions*



The results showed that ordering and comparing fractions errors are systematic in nature, thus conceptually driven not incidental. The very high error percentage of whole number bias (29.5) indicated that even when faced with fractional quantities, the students still applied their previous knowledge of natural numbers in favour of a higher frequency with whole-number bias. Denominator misinterpretation (14.0%) and numerator-only comparison (15.5%) indicated fragmented understanding of the relationship between the numerator and denominator in fractions. These results are consistent with reasoning that learners focus on numerical features in isolation (Siegler & Pyke, 2013) rather than interpret fractions as multiplicative relationships between two quantities. Students seem to judge fractions as independent whole numbers, never relating numerators with denominators — up until October of 2023. An example of this misconception is that it gives rise to reasoning without regard to the relational meaning of fractions, giving priority to numerators (Ni & Zhou, 2005). This is a phenomenon well recorded in the literature where it has been described as an overextending of whole-number reasoning which prevents reasoned thinking about rational numbers.

Moreover, the pervasive nature of errors across all sub-topics indicates that: students do not have a conceptual flood of fractions. Recent studies have shown that learner misconceptions are occurring regularly while taking protosections considering the numerators and denominators as independent integers, such of which result in systematic errors during a sequence of fraction operations (Kukulaje et al, 2024) Siegler & Lortie (2014) found that whole-number thinking persists in rational learning.

This dominance of this error is critical, as it implies that it is likely a primary error upon which many of the other errors stem.

The significant frequency of errors in ordering several fractions (20.4%) and lack of fraction magnitude sense (18.3%), again, indicates that students have difficulty thinking of fractions as quantities with measurable size. Instead of thinking about size, learners too frequently use less valid surface level mechanisms such as comparing numerators and denominators of fractions. Fraction sense has been found across multiple empirical studies (e.g., Fazio et al. 2016) to be foundational to overall mathematics mastery, and a strong predictor of later achievement. Most recently, literature reinforces that magnitude understanding is not an automatic outcome of procedural directions (Fisher & Dennis, 2023; Gomez et al., 2023); but instead requires explicit conceptual work often aided through representations—for example, the use of fraction bars/other visual C—to learn how to compare and calculate in math. So, it is no surprise that students are likely to make a high percentage of these errors, indicating their experiences with learning about fractions do not provide adequate practice in conceptual representations.

Furthermore, errors associated with denominator misinterpretation and reversal reasoning indicate an inherent weakness in students understanding of fraction size.

The successive errors made in the test show that these difficulties are not restricted to a single type, but pertain to most types of fraction operation. The trouble students have comes in when the problem is more involving, that every component of a task has been solved. Handling multiple denominator issues, as well as making sense of numbers in a combined form may have led to inconsistent performance. Similar results have been reported in recent research, which indicates that students often struggle with fraction arithmetic due to an inappropriate application of whole-number bias (XU et al., 2024).

Furthermore, procedural errors such as incorrect cross-multiplication (8.5%), confusion with the same numerators or denominators (16.8%) indicate strong evidence of an imbalance between procedural fluency and conceptual understanding. Although some students appear capable of applying learned concepts, their inconsistent execution and inability to adapt these procedures suggest that such knowledge is superficial.

Conclusion

These findings are consistent with research indicating that procedural competence without conceptual understanding often results in inflexible, misconceptions, and error patterns (Hiebert & Grouws, 2007). The coexistence of multiple procedural errors suggests that students are not only forgetting the rules but executing the concept erroneously due to less or incomplete understanding of the concept.

Further evidence from large-scale research indicates that students' errors in fractions can be characterized into consistent patterns. These recurring patterns demonstrate that there is an organized character to misunderstandings, which are normally related to trouble with the representation and execution of procedures (conversion, simplification or partitioning); Miller & Lorrain, 2024; – Caute, 2026. The consistency of these findings with earlier and recent literature suggests that students who struggle with fractions do so as part of a larger problem learning to construct a cohesive and flexible understanding of this topic. The student's performance conforms to a general trend of poor accuracy with regard to fraction operations, corroborating the large-scale assessment report from Punjab Examination Commission, Pakistan (2023). Notably, the fact that different error patterns tend to co-occur suggests that these difficulties with fractions are not just independent

problems. Elliott et al findings suggest a chain of causation: Whole-number bias leads to misunderstanding magnitude, which causes ordering and comparing error on fractions. This is a good fit with the view that misconceptions in mathematics are widely organized and entrenched, only being displaced when challenged by appropriate instructional actions.

Thus, the overall results indicate that misconceptions and errors in fractions arise from a complex interplay of cognitive biases, misinterpretation of concepts, lack of learning framework or sequencing instruction correctly. The prevalence of whole-number bias and the lack of consistent ability in using magnitude reasoning and appropriate procedural application to concepts suggests that existing classroom instruction is limited in helping students develop conceptual understanding of the fraction concept.

Hence, there is an urgent requirement for pedagogical interventions that focus more on conceptual understanding with an appropriate blend of multiple representations strategies and proactive confrontation of misconceptions. These types of approaches not only are an important support for understanding fractions, there close to essential to achieving long-term mathematics success.

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